

MATH 122B AND 125  
ADDITIONAL REVIEW PROBLEMS

1. Let  $f(x)$  be a continuous function such that  $f(1) = 3$ ,  $f(7) = 6.059$ , and  $f'(x) = \ln\left(\frac{x}{x^2 + 2}\right) + 2$ .

a) What is the domain of  $f'(x)$ ?

b) Find  $\lim_{x \rightarrow \infty} f'(x)$ .

c) Find  $\lim_{x \rightarrow 0^+} f'(x)$ .

d) For what value(s) of  $x$  does  $f(x)$  have a horizontal tangent line (if at all)?

e) Is  $f(x)$  invertible on  $(0, \infty)$ ?

f) For what value(s) of  $x$  does  $f(x)$  have a local minimum (if at all)?

g) Does  $f(x)$  have a global maximum on the interval  $[9, 12]$ ? If so, what is the  $x$ -coordinate?

h) Find the equation of the tangent line to  $f(x)$  at  $x = 1$ .

i) If we restrict  $x$  to the interval  $(1/2, 5)$ , find  $(f^{-1})'(3)$ .

j) Does  $f(x)$  have an  $x$ -intercept to the right of  $x = 7$ ?

k) Does  $f(x)$  have a horizontal asymptote?

l) Find  $f''(x)$ .

m) Find  $\lim_{x \rightarrow \infty} f''(x)$ .

n) For what value(s) of  $x$  does  $f(x)$  have an inflection point (if at all)?

o) Over what interval is  $f(x)$  concave up?

p) If your line in part h) is used to estimate  $f(1.2)$ , would we get an under or over estimate of the true value of  $f(1.2)$ ? How do you know?

q) Find the exact value of  $\int_1^7 f'(x) dx$ .

r) Find the exact value of  $\int_3^5 f''(x)dx$ .

s) Use the Fundamental Theorem to estimate  $f(5)$ .

t) If we estimated  $\int_2^4 f'(x)dx$  using a left hand sum with 20 subdivisions, would we get an under or over estimate of the true value of the integral? How do you know?

u) If we estimated  $\int_2^4 f(x)dx$  using a left hand sum with 20 subdivisions, would we get an under or over estimate of the true value of the integral? How do you know?

v) Find  $h'(x)$  if  $h(x) = \int_1^x f'(t)dt$ .

w) Let  $g(x) = (f(x))^3$ . Find  $g'(7)$ .

x) Suppose  $f(x)$  is the temperature of some object in degrees when  $x$  is measured in hours. What are the units of  $f'(x)$ ? Find  $f'(3)$  and give a practical interpretation.

y) Suppose  $f'(x)$  is measured in degrees per hour. What are the units of  $\int_1^4 f'(x)dx$ ? Give a practical interpretation.

z) That's all.